Abstracts of Contributed Talks of the CADGME 2016 Conference

1. **Vanda Santos, Perdo Quaresma**

Adaptive Strategies in the Web Geometry Laboratory

The Web Geometry Laboratory (WGL) platform is a collaborative and adaptive blended-learning Web-environment for geometry, it integrates a dynamic geometry system (DGS), it provides a collaborative environment for students and teachers and provide some adaptive features. Its use is possible in the context of a classroom or remotely (hilbert.mat.uc.pt/WebGeometryLab).

To be able to build individual students profiles and/or individual learning paths, the system collects geometric information about the students' interactions when in the regular, stand-alone, mode. In the regular mode all the steps done be the students when using the DGS to complete a given task are recorded.

An initial case study in Portugal was done, sustained through a qualitative approach, with a 15 years old student in an adaptive environment context. The teacher was able to see the work done, from the first step to the last step, step by step (in a faster speed or slower or to pause), analysing the steps done by the student to solve the task, getting information that can be used to assert the student's van Hiele level.

The development work currently being done regards the construction of a learning path editor. This editor will allow the teacher to build learning paths differentiated by students' profiles. These profiles will be build, by the teachers, using the information collected previously by the system. Each learning paths will be a (non-)linear sequence of tasks to be solved by each individualised student.

The WGL platform will include in future stages of development a geometric automated theorem prover to be used in the automatic, or semi-automatic construction of the students profiles, and in the learning process.

2. **Zoltán Kovács and Csilla Sólyom-Gecse**

GeoGebra Tools with Proof Capabilities

We report about significant enhancements of the complex algebraic geometry theorem proving subsystem in GeoGebra for automated proofs in Euclidean geometry, concerning the extension of numerous GeoGebra tools with proof capabilities. As a result, a number of

elementary theorems can be proven by using GeoGebra's intuitive user interface on various computer architectures including native Java and web based systems with JavaScript. We also provide a test suite for benchmarking our results with more than 200 test cases.

8. **Anatoli Kouropatov, Regina Ovodenko and Sara Hershkovitz**

The impact of digital tools on students' learning of geometry

The integration of digital technology into the mathematics classroom is an ongoing process (Laborde & Sträßer, 2010, p. 122) which follows the national tendency in different countries. The professional literature indicates that combined mathematics teaching technology tools help in the process of constructing an abstract knowledge of mathematics, and geometry in particular (Lagrange, J.B. et al., 2003). Based on this idea, the Center for Educational Technology (CET) developed a one-year course devoted to the subject of studying geometry using an interactive online geometry environment. The observed students' work and performance were varied and interesting.
In the proposed working group we suggest to focus on students' learning of definitions.
During the course, we observed that students' learning process towards constructing concept definition includes the following stages: initial experience to defining, attempting to define well, and using the definitions in problem solving (including proofs by contradiction) with the purpose of consolidating the concept.
Based on these observations and on the gathered empirical evidence, we can raise several questions for the working group to discuss regarding the impact of digital tools on students' learning of geometry as follows:
1. How does the process of concept definition construction occur in a student's mind? What is the cognitive path from concept image to concept definition?
2. What are the tools students use on the path and what are the stages they pass in an interactive dynamic environment?
3. Should we distinguish individual knowledge and shared knowledge? If yes – how?

10. **Mohamed El-Demerdash, Pedro Lealdino and Christian Mercat**

The Effectiveness of Kinesthetic Approach in Developing Mathematical Function Graphs Recognition and Understanding at University Level

This research paper investigates our belief as educators that kinesthetic approach can be used to develop students' recognitions and understanding of mathematical function graphs with the help of a competitive dancing game.

Given that some students need to use other senses as a preparation to abstract and symbolic thinking, we developed Augmented Reality kinesthetic digital resources around standard mathematical function graphs using Microsoft Kinect sensor and some programming of Unity 3D Development Kit, Microsoft Kinect (SDK) - Software Development Kit - and CindyScript the programming language associated with Cinderella - Dynamic Geometry Software, linked through a UDP (User Datagram Protocol) connection. The software recognizes students’ body gestures as input representations for mathematical function graphs.

Aiming at verifying the resources effectiveness in developing recognition and understanding of mathematical function graphs among freshmen students at UCBL1, a quasi-experimental design with pretest and posttest is considered. The test consists of 40 items that are equally distributed among the two levels of achievement: recognition and understanding. 20 items are designed at each level in isomorphic pairs that are administered randomly as whether pretest or posttest. The items type for the test are multi-choice questions of 4 choices. The test items were written in verbal and nonverbal, symbolic and graphical ways.

The experiment with freshmen students is going to be implemented at the beginning of summer semester 2015-2016 at INSA de Lyon- UCBL1. The experimentation is intended to be conducted as a challenge where volunteer teams of freshmen students will have to compete by achieving highest scores and be rewarded by some goodies. The learning gain represented in students’ recognition and understanding using the prepared achievement test is going to be computed and correlated with the scores in performing the kinesthetic game in order to look for evidence of a relation between their engagement and achievements.

12**. Péter Körtesi**
Using GeoGebra to study the Famous Curves of the MacTutor History of Mathematics archive

We will study the Chapter Famous Curves of the MacTutor History of Mathematics archive, see: http://www-history.mcs.st-and.ac.uk/Curves/Curves.html. The GeoGebra softvare is suitable to represent both the set of functions, and the so called associated curves, like evolutes, or involutes, and to experience their relation. The curves are given either in explicit, implicit, parametric or polar coordinate form, and we will explore the power of the softvare to visualize them. The osculating circle, tangents, normals, convex boundary of family of curves or Taylor series will be mentionned as well. The above mentionned Famous curves chapter is suitable as well to offer practical examples for students in applying GeoGebra, and their results can be valourized on the GeoGebra Tube, some of examples:
https://www.geogebra.org/material/simple/id/1226675
https://www.geogebra.org/material/simple/id/1222515

https://www.geogebra.org/material/simple/id/jQNWkzhG
http://www.geogebra.org/m/CAm5xR6s

13. **Gregor Jerše and Matija Lokar**

Learning and teaching programming and numerical methods with a system for automatic assessment

In the curriculum of the college level program "Practical mathematics" a lot
of emphasis is put on learning programming as well as on using acquired
programming skills in various branches of mathematics. Numerical methods are
naturally one of them.

Programming is a skill that is best learned by practice and numerical methods
also require a significant amount of practical examples. So teachers are
required to expose students to numerous problems and supervise the students'
attempts to solve them. To support this teaching approach we have developed a
web service called Projekt Tomo.

The service is designed to require little or no additional work from students

and teachers, enabling them to focus on the content. The service works as

follows: a student first downloads files containing problem descriptions to
his computer. The files are opened in his preferred development environment
and the student starts filling in the solutions. Executing the files checks
his solutions locally. If server is available the solutions are automatically
stored and optionally verified on the server.

This approach has several benefits: the service provides instant insight into
the obtained knowledge to both student and teacher, all without disturbing the
teaching process. There is also no need for powerful servers since all

executions are done on a student computer.

Currently the system supports programming in Python (with the NumPy library)
as well as programming in GNU Octave, a language quite similar to Matlab.

15. **Miguel A. Abanades, Francisco Botana, Zoltan Kovacs, Tomas Recio and Csilla Solyom-Gecse**

Automatic Discovery in GeoGebra: First Steps

Automatic discovery of geometric facts is a manifold concept. Basically, it refers to methods for finding complementary hypotheses for a conjectured statement to become true. Say, one conjectures that the projections of a free point P, on the sides of a given triangle, are collinear; then we would like to automatically find out that this is true iff P lies on the outer circle of the triangle. As this example shows, automatic discovery includes a generalized, implicit locus computation (the locus of P). On the other hand, automatic derivation of formulas (say, finding the relation between the radii of the inner and the outer circle of a triangle and the distance between their centers, ie. Euler’s theorem) can be considered as a sort of automatic discovery in which we consider a trivial thesis and we look for new hypotheses, just involving the variables describing the two radii and centers’ distance.

In our talk we will present our current work on developing tools for the automatic discovery and derivation of elementary geometry statements, based on the implementation of computational algebraic geometry methods onto the dynamic geometry program GeoGebra. It is, in our opinion, the first time such tools will be available on a widely disseminated dynamic geometry program, allowing it to automatically (and wisely!) guide a student through an inductive exploration process on a geometric context.

The emphasis in our "Justifying (in) Math” presentation will be placed on two issues: 1) the contradictory, unexpected, results in some simple cases, requiring the implementation of a subtle protocol in our discovery algorithm, 2) the diverse mathematical and educational challenges posed by this -apparently straightforward- application of some well known algorithms onto the GeoGebra framework.

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16. **Norbert Bogya**

Playing card game with finite projective geometry

High school students learn the standard Euclidean geometry. Although they have a lot of applications, neither projective geometry nor finite geometries are included in the general high school curriculum. In this talk, we give an elementary introduction to finite projective geometry with the help of a card game, called Dobble. It is not a traditional logical game, it is based on speed, and players need good reflexes. This game almost perfectly models PG(2,7), the finite projective plane of order seven and gives us a playful approaching method. We investigate its most important properties as well as we consider the possibility of constructing similar cards based on finite projective planes, or other ways. We also discuss the problems arising at the constructions. We show some algorithms and demonstrations that help to generate such card games in Wolfram Mathematica.

18. **Eleonóra Stettner**

GEOMATECH COMPETITION

The GEOMATECH project includes not only teachers' training and the development of teaching material, but also national competitions for Hungarians living in and outside Hungary. We prepared tasks for six different age groups, between 6 and 18 year - old students. Groups of 1 – 5 persons could participate in the competition. There were competition rounds for 9 months, and each round had a central topic.
Problems that the compilers of the tasks had to solve:

- We had to set tasks which gave work to think about and to create enough for one month for the whole group.
- The tasks had to allow everybody to join in, not only the best mathematicians, but they had to contain enough challenge for the best ones, too.
- The tasks had to be new and interesting so that students feel the urge to enter.
- The use of GeoGebra had to be essential to the solution of the tasks.

We asked the upper primary school and the grammar school students to keep a diary. Looking back it proved to be a very useful idea, although it wasn't easy for the children to formulate the explanation and the discussion. But on the one hand, it was useful because this way they are getting used to write down their thoughts, and on the other hand, we received a lot of useful comments in connection with the competition and with GG. Moreover, from the diaries we can see that while understanding and solving the tasks, the children had to cooperate, argue, communicate and help each other.
In my presentation I would like to speak about the experience of the competition and to show same interesting and beautiful solutions.

20. **Walther Neuper**

Engineering mathematics -- intuitive and formal

Systems based on technology from (Computer) Theorem Proving (TP) excel at formal rigor, at reliably and generously checking user input and at ensuring correct results -- advantageous features for educational systems. Isac, a prototype of such a system, additionally supports stepwise problems solving by use of Lucas-Interpretation.

Isac's conception works out nicely in "pure mathematics", where problems are formally specified. Here the whole process of problem solving can be covered: the model-phase, the specification-phase and the solve-phase (more about these phases below). In all three phases the student gets support in stepwise interaction similar to traditional paper-and-pencil work; support means reliable check of input and "next-step-guidance" (the systems suggests a next step if the student gets stuck).

However, physicists and engineers generally follow a more intuitive and less formal approach --- now, how can a system like Isac cope with these requirements? The talk will illustrate ideas by examples:

Model-phase (aims at a formal model of the problem at hand): Engineering problems frequently are described by use of figures depicting forces, torques, directions, etc. The idea is to make respective figures interactive, such that forces, distances, etc. can be interactively attached at certain places.
Specification-phase (relates model and sub-problems to knowledge mechanised in the system): sub-problems shall be moved into the electronic worksheet, arranged and connected, while Isabelle/Isac's type system helps to check the connections.
Solve-phase (interactively construct steps towards the result): So far all steps have been formally justified by some theorem (or a combination of theorems). The idea is to extend these justifications by lines of prose. How these shall be handled and relate to the requirements of TP is under

consideration.

The ideas mentioned above are pursued in a R&D-project in cooperation with
Universities of Applied Sciences in Austria.

21. **Martin Günzel, Tereza Suchopárová and Helena Binterová**

Tessellations in lower secondary school classes

In this article the authors want to present a project that was realized at their lower secondary school. The aim of the project was to use inquiry based learning to introduce tessellations to the pupils. This article presents the materials that were created for the project together with teachers experience from the actual lessons. The first lessons were focused on students' motivation and also revision. Pupils recapitulated or learned some important facts from geometry that are necessary to understand and work with tessellations. In the second part of the project, pupils were instructed to create three versions of a tessellation: a traditional construction, a computer construction and an actual model that would cover their desk. The criteria for the following evaluation of pupils' work were: use of mathematical knowledge, originality, accuracy, beauty and team-work. During the project pupils had to work individually but at the same time as a team. They had to observe, learn and use their knowledge to complete the goal. The results of their work are presented in this article as well.

22. **János Karsai, Zsolt Vizi, Eszter Szénási and Lőrinc Pósfai**

Modeling approach in teaching math students

On the previous CADGME conference, we spoke about the problems arising in teaching math courses for students in applied sciences. Now, it turns out, we need to speak about teaching applications to math students. Experimental arguments are accepted and even preferred in applied sciences, and such approach is also useful in theoretical mathematics, but it is not easy to convince the students about the use of experimental and modeling approach.
Although the interdisciplinary and modeling approach became popular "keywords" in teaching mathematics students, the current practice is not as nice as we should think even in 2016. Deep theories are learned by the math students with hardly any applications. A new course "Mathematical model" given to freshman students try to help to resolve this problem.
In our talk, we will present the main idea, didactic goals, the curriculum and experiences of teaching models to our math students. We illustrate the talk with a lot of examples and dynamic demonstrations prepared in Wolfram Mathematica.

24**. Matija Lokar and Paul Libbrecht**

Obstacles in combining the use of various tools in solving mathematical problems – why is Copy/Paste often useless

In first cycle professional study program Practical Mathematics at Faculty of mathematics and physics, University of Ljubljana, a course entitled Computer tools in mathematics is proposed. Its syllabus is primarily focused on approaches towards solving mathematical tasks with a computer, mostly with the programs for numerical and symbolic calculations, such as Wolfram's Mathematica, Matlab, GeoGebra.

As mathematical software is diverse and rich and each software can perform some tasks very well and others only with great efforts, we decided to use various tools, and not to focus on a single one. This included also tools to create learning content such as word processors. We envisioned a scenario where students work with several systems, exchange the mathematical objects from one to the other and use mathematical objects in reports. Students naturally expect to do this using the copy and paste functions. However, such exchanges are often bumpy: special copy functions, plain-text adjustments, or careful verification must often be used.

The students, and we, explored transfers of mathematical objects between various programs using copy and paste: the expectations of the students and their observations are reported. The experiments show a multitude of issues. It appears that copy and paste for regular desktop users is not yet the object of mainstream testing in mainstream tools to create content. However, many technical possibilities exist and seem to be promising.

25. **Paul Libbrecht and Matija Lokar**

Expectations of the Copy and Paste Action for Formulæ

When an object, of any nature, is displayed and selectable on a computer screen, users expect it to be copy-and-paste-able: one can invoke the copy function and insert (paste) it at other places, within the same programme or beyond. Operating systems have been offering APIs to this effect since decades and many successful copy-and-paste operations are part of the daily routine: texts and images, at least. For mathematical objects, this is not so: geometric constructions of dynamic geometry systems or mathematical formulæ displayed within various computation or editing systems could, in principle, be transferred, but fail often to be so.

This presentation describes the principled expectations about copy-and-pasting mathematical formulæ in a learning context. For example for students it is natural to expect to use a web browser to read a document but to use a computing system to perform calculations on the formulæ of it. Or perhaps to take the solution provided by Mathematica and insert it into the report, prepared by Word or Libre Office Write.

Having chosen four widespread software programmes, we have explored transfers between them using copy and paste. The experiments we describe show a multitude of issues, and explain some of them. It appears that copy and paste for regular desktop users is not yet the object of mainstream tested functions even though quite many technical possibilities exist.

To nail this further, we attempt to classify the expectations of users wrt to copy-and-paste: which ones might be realizable provided software adjustments, and which ones are doomed to fail, because of an essential difficulty.

26**. Masataka Kaneko and Setsuo Takato**

Collaborative use of KeTCindy with CAS

Manipulating mathematical objects on a PC screen interactively and paper and pencil-based activities to process mathematical reasoning are both important aspects of mathematics education. In order to establish an effective linkage between these two sorts of activities, we have developed a plug-in named KeTCindy into the excellent dynamic geometry software Cinderella. KeTCindy converts the procedure of drawing graphical objects on the Cinderella screen into TeX readable code to generate the corresponding mathematical artwork in its final PDF output. Thus, KeTCindy makes Cinderella an efficient graphical user interface for generating high-quality mathematical artwork on TeX documents. Also KeTCindy extremely enhances the editing and graphics environment of TeX.

To improve graphics capabilities of KeTCindy system, using it collaboratively with the symbolic computation capabilities of various computer algebra systems (like Maxima, Scilab, Risa/Asir) or well-structured simulation capabilities of statistical software (like R) is desirable. Therefore, we have implemented the function of invoking those computing software programs and importing the data calculated or simulated by using them into KeTCindy. Combining these imported data with the interactive graphics capability of Cinderella should result in the possibility of presenting extremely wide range of mathematical objects.

In this talk, we will show some basic functionalities of KeTCindy system together with some sample class materials generated with it. While showing them, the merits of the above mentioned collaborative use will be emphasized.

27. **Přemysl Rosa and Vladimíra Petrášková**

Potential of Maple as tool for improving financial education of future teachers

In the Czech Republic one of the key tasks of an education system is to ensure the financial literacy of citizens. Scarcely any topic of the school curricula is undergoing such rapid and frequent changes as the topic of financial issues. Therefore its teaching requires new approaches that would dynamically respond to the current situation. The contribution deals with an original collection of educational materials developed by the authors to support the financial education of future teachers at the Faculty of Education at the University of South Bohemia. In addition to the usual computer means of financial computation such as spreadsheet and online calculators the authors discovered the utilization of the computer algebra system Maple to be beneficial. Maple enables to create interactive ”smart documents“ whose interactivity consists in implementation of a simple user interface beyond the framework of usual document. This fact enables the user to influence the computation result by a change in input parameters and thus de facto to simulate an inexhaustible number of situations. These documents can be extended by executable applications programmed directly in program Maple, maplets. The main benefit of these applications is that even if they utilize the whole computation potential of program Maple, their user environment can be limited to required functions and thus it is possible to reach a close specialization of particular applications.

28. **Jiri Blazek and Pavel Pech**

Computer-aided investigation of sets of points in geometry

The contribution deals with (partly open) geometric problems in 3D space, which are mainly investigated by means of algebraic software like CoCoA, Maple or Singular. On a model example it is demonstrated how current computer programs may be incorporated into solution process of various geometric problems, among others how conventional synthetic approach may be supplemented by computer-aided methods. We also shortly discuss DGS programs and their possibilities in the process from formulation a hypothesis to its verification or disproval. Finally, we mention history of some famous geometric problems and their transformations and reformulations caused by CAS and DGS programs appearing in 1990s.

29**. Štefan Berežný, Kristína Budajová, Eva Komová and Henrich Glaser-Opitz**

The MATH and the Vernier System at Faculty of Aeronautics

This article describes two interesting and educational challenges whose we prepared for the students at faculty of Aeronautics, Technical University of Košice. The first is a software application MATH[4] used to teach Applied Mathematics, with focus on Numerical Mathematics and the second is the Vernier system, which we used in the Physics student’s laboratory.

30**. Valentyna Pikalova**

Teaching and Learning Math Behind Computer Science with the Help of GeoGebra and Python

The paper is focused on topics where mathematics and computing are most relevant to each other, emphasizing the bridges between theory and practice. Topics include sets, relations, elementary graph theory, asymptotic notation and growth of functions, permutations and combinations, discrete probability. The main tools in order to support and

compare problem-solving technique are GeoGebra and Python.
The main goal is not only to combine an appreciation of mathematical reasoning with an understanding of computing but enrich both of them through interdisciplinary approach.

31. **Denys Stolbov**

Visual models of cipher algorithms for students' learning information security

Our long-term experience of teaching students the basics of information security shows that a cryptography as a chapter of this course is not easy to learn. The students have problems with understanding some encryption algorithms, especially their complicated mathematical tools. On the other hand, it is quite difficult for a teacher to explain for students main features of structure and functionality such algorithms. At the same time, it has been found experimentally that visual information is perceived and remembered by a person better than text, sound and tactile. All this stimulated us to develop special dynamic computer models to visualize several cipher algorithms. In our study we represent the models, which describe and demonstrate the main stages of the public key (asymmetry) encryption algorithms. Firstly, in one of our models we rendered a mechanism of generation a pair of algorithm's keys and a procedure of their exchange between a sender and a receiver. Secondly, we paid attention to the visualization of RSA algorithm. An asymmetry of this algorithm is based on the practical difficulty of factoring the product of two large prime numbers, the factoring problem. Our implemented computer model of RSA algorithm allows to present complicated mathematical idea of the factoring problem in a simple, understandable and accessible form for the students. The models were implemented by us in dynamic mathematics software GeoGebra that is open source with powerful tools for quickly and easily created of computer mathematical models.

32**. Zlatan Magajna**

Technology as a support for generating and presenting proofs in geometry

Dynamic geometry and similar software is a constitutive part of today's school geometry. However, when proving and learning about proofs in geometry is considered the role of technology is currently marginal, and the future role, due to the promising developments in automated theorem proving, is yet to be set. Regarding proofs and proving in school geometry the didactic focus is currently not on automated proving, but on issues like: whether and how students understand the concept of proof, how to present proofs to students, how students understand proofs of theorems, how to assess their understanding of proofs, and how to enable students to produce their own proofs – and how to profitably and effectively involve technology in treating these issues. Several questions are still open, e.g.: Should technology direct students to paragraph form, flowchart or two-column presentation of proofs (or to all three of them)? Should software tools emphasize symbolic, verbal or static/dynamic visualisation or all such presentations?

We shall present some tentative solutions to the above questions as are implemented in OK Geometry software and are not found in common dynamic geometry software. Most of the solutions are simple aids to be used in generating proofs of geometric statements (e.g. automated observation of properties), some of them are tools for writing up proofs, and some of them are designed for presenting proofs. Such software tools have little impact if they are not incorporated in some method for learning proving. For this purpose we shall present two approaches. The first one is a simple method of producing a flowchart proof by ordering pictorial representations of properties. The second method is a three step method of generation/presentation of proofs. The method is derived from the assessment model of Young and Lin and, we believe, is suitable for novices and less able students. We also plan to present a pilot study on the effectiveness of the second method.

34. **Ildikó Perjesi-Hámori and Csaba Sárvári**

More or less? Using CAS in Mathematics teaching based on 15 years of experience

In our talk, we will summarize the experiences from our 15 years since begin using Computer Algebra Systems in teaching Mathematics at the University of Pécs, Hungary. In the beginning (let's call it experimental period) we tried to examine the usage of CAS from a didactical point of view (multiple representation, modularization, internet as a new tool, experimental learning in CAS environment, problem solving in CAS environment. During the subsequent period we used a pre-designed worksheets during the entire semester, which were distributed to students in the beginning of each semester. In this period, we (the teachers) discovered the possibilities of this new tool and tried to use it all the time. In the third period (currently), CAS is used not only in Maths courses at our Faculty, but also in several other courses such as cryptography and physics. We learned the didactical limits of CAS. During practical lessons students usually do not receive prepared worksheets, but only some assistance from teachers. So that, students solve problems alone under the supervision of teachers.

We will offer some examples how our way of teaching was changed throughout these periods due to the development of software and hardware; the results of our didactical experiments; and the new possibilities of blended learning. We show some observations to these questions: What is the difference between teaching students with different cultural backgrounds and preliminary knowledge? How can we teach small groups as well as large number of students?

35. **Rein Prank, Evari Koppel, Joosep Kibal, Katrin Valdson and Joosep Norma: Word Problem Solution Environment TEKSTER**

TEKSTER is an environment for solution of word problems by finding at each solution step a value of some sensible quantity. Such tasks are solved in Mathematics of elementary grades but also in Geometry, Physics and Chemistry.

At each step the student performs the following substeps:

1) Using given phrases to build a question/sentence that tells what quantity will be calculated at this step;

2) Forming from initial data and results of previous steps a numeric expression that finds the desired quantity (using arithmetical operations);

3) Calculating the value of the expression.

At the first substep the program checks whether the sought quantity belongs to the ‘reasonable’ intermediate results specified by the author of the task and whether it can be found using the allowed number of operations. The equality between the student’s and teacher’s expression is checked at the second substep and the correctness of calculation at the third step.

The program uses the Maxima computer algebra system for checking equality of algebraic expressions and the automated Solver for checking the solvability of tasks, for demo solutions and generation of hints. The student program records in solution file the choices and mistakes made by the student.

In task composition environment the teacher types the text of the task, marks the numbers that can be used in solution, composes the matric(es) of phrases for questions, and inputs the formulas that express the intermediate results through initial data. The program can generate numerical values of initial data at random. The presentation demonstrates both student and teacher environments.

36. **Štefan Berežný**

Implementation of Research Findings in the Laboratories of DMTI

We have adapted our two laboratories on our department based on our research results, which were presented at the conference CADGME 2014 in Halle and published in the journal [1].

We describe the hardware and software structure of the Laboratory 1: LabIT4KT-1 – Laboratory of Computer Modeling, and the laboratory 2: LabIT4KT-2 – Laboratory of Numerical Mathematics. We describe the functionality of these laboratories in teaching mathematics courses at our faculty. The main idea is to give priority for usage freely available software.

We focus mainly on the following mathematical subjects: Operational Analysis, Linear and Quadratic Programming, Numerical Mathematics, Applied Statistics, Queuing Theory, Software Computing Resources, Software tools for modeling processes, Applications of Differential Equations, Optimization Methods, and Fundamentals of the LATEX.

38. **Norbert Bogya, Lajos Szilassi and Zoltán Kovács**

Euclid, Bolyai and the exemplification in teaching of geometry

After a brief historical overview we show how we can illustrate the most basic statements of hyperbolic geometry with modern computing techniques. To do this, we use GeoGebra.

A lot of interactive experiments, including GeoGebra files, can be found on the Internet, and these programs try to illustrate the most basic statements and definitions of the hyperbolic geometry with the help of the Poincaré disk model.

The model presented by us differs from them, because it gives the users an own, precisely created toolbar, and a list of task to introduce, use and practise them. Across a didactically well-structured problem series we show the connection between the well-known Euclidean geometry, the less-known hyperbolic geometry and their common part, called absolute geometry. We concentrate on their objects, basic definitions and statements.

39. **Ruti Segal, Avi Sigler and Moshe Stupel**

Problem Posing and Problem Solving of Geomatrical Configurations by Integrating Dynamic Geometry Software

Our purpose is to describe research of prospective teachers using a geometrical configuration, which was carried out with the WIN (“What if Not”) method by integrating dynamic geometry software. The prospective teachers integrated problem posing and problem solving, handled “prove” and “find” problems as recommended by Polya. The vast majority of the prospective teachers reported that they “are doing math”, and as Brown & Walter mentioned, they perceived themselves as participants rather than spectators. Most of the prospective teachers recommended integrating courses dealing with WIN inquiry to train mathematics prospective teachers as well as presenting it in the high school curriculum, in order to raise motivation and to deepen the knowledge pool of learners.

40. **Victor Oxman, Avi Sigler and Moshe Stupel**

“What if not” investigation method with the aid of Geogebra of a geometric configuration of quadrilaterals that through a dynamic process aspire to be square

The purpose of this article is to describe the building of a geometric configuration and explore it, in order to train students in teaching mathematics according to:
‘What if not?’ (WIN) strategy suggested by Brown & Walter (1990).
The principles of the construction of the configuration are:
1. It has a generalization potential .
2. It has elements of “prove” and “calculate”.
3. Challenging but not frustrating.
4. Allows construction of hypotheses which can be checked with Geogebra.
5. Thoroughly researched by us to prevent divergence or dead ends.
The chosen configuration begins with a rectangle whose exterior angles bisectors, when extended, form a new quadrilateral.
By using the WIN method (“what if not a rectangle”), different external quadrilaterals are formed, with a common characteristic that the student will quickly discover and with the aid of Geogebra, immediately prove.
The investigation branches off in directions of proving, calculating and extreme values, up to the question of the path of the ray of light in the external quadrilateral. We, who built the configuration, explored many patterns, and the interesting and surprising mathematical results are included in the article.
We also built a dynamic process that starts with a quadrilateral, continues to form another quadrilateral with its exterior angles bisectors extended, and from this form continues to create other quadrilaterals with its extended exterior angles bisectors. For example: a parallelogram forms a rectangle, which forms a square, which forms another square, where the process ends.
When this dynamic process is applied on different quadrilaterals, what emerges is a series of quadrilaterals that monotonically aspire to a square, in the sense that their angles all aspire to become 90 degrees, including the angle between the diagonals.
The creation of the exploratory configuration in combination with the technological tool, benefits the student, in our opinion, by giving him the

sense that he is not only learning mathematics, but also doing mathematics.

41. **Avi Sigler, Victor Oxman and Ruti Segal**

The development of interesting connections between the radiuses of circles that are inscribed in or by triangles, and the discovery of unique features, with algebraic manipulations and dynamic exploration.

The discovery of connections between the radius of a circle inscribing a triangle, a circle inscribed by a triangle, the radiuses of the inscribed circles outside the triangle (tangent to the outside of one side and to the continuation of the other two sides), and the connection between them and the length of the triangle sides, and some parts of the triangle, is a fascinating topic, which can be investigated by raising hypotheses and investigating them dynamically with the computerized technological tool, and also with formulaic mathematical proofs, with a use of various mathematical tools, assisted by algebraic manipulations.
Since Leonhard Euler discovered in 1765 that there are 9 points in a triangle that intercept with a single circle, and the discovery in 1822 by Karl Wilhelm Feuerbach that this circle is externally tangent to the triangle's three excircles, mathematicians have been interested in finding connections between radiuses and different sections.
As part of an advanced course for math teachers on the topic of the combination of fields of mathematics, the students were asked to investigate the question: Do the Euler circle and the triangle's incircle intersect, are tangent to each other, or unconnected?
At first, the issue was investigated dynamically by the Geogebra software, and when a positive answer was given, the students had to prove it mathematically, using known formulae derived from many sources.
Subsequent to that, the student were asked to prove the following connections:

(1) $1/r=1/Ra+1/Rb+1/Rc$
(2) $Ra+Rb+Rc>=9r$
(3) $P/S=1/h1+1/h2+1/h3=1/r$
(4) $Ra+Rb+Rc=4R+r$

The triangle ΔABC has sides a, b and c. Its incircle has a radius of r. Ra, Rb and Rc are the radiuses of the external circles tangent to the sides a, b and c respectively, and tangent to the extension of the triangle sides. P is the half the circumference of the triangle ΔABC. ha, hb, and hc are their heights.
Another task investigated was:
Formula of sum of the distances from the center of the triangle circumcircle to the sides of the triangle.

Triangle ΔABC, which is not obtuse, is inscribed in a circle (O, R). Lines are drawn from the center of the circle O to the sides of the triangle: h1, h2, and h3.
This is true: h1 + h2 + h3 = R + r, when r is the radius of the triangle inscribed circle.
Does this characteristic apply for every triangle?
The following points were studied methodically as part of the activity:
• The level of assimilation of the technical tool among the students are the probability of reuse.
• The students’ ability to combine tools and different mathematical fields in order to achieve proof and carry out research.
• The level of contribution of research tasks to the students’ mathematical knowledge.

42. **Satoshi Yamashita**

Producing Class Materials with KeTCindy ― Programming Styles, Creating Portal Site and the Evaluation

To produce class materials with figures using TeX, the KeTpic Development Group (KDG), comprising S. Takato, the author and several Japanese mathematics education researchers, completed KeTpic in 2011 as a plug-in for the Scilab numerical analysis software package. KeTpic users produce KeTpic programs based on their original programming styles. This leads other KeTpic users to a shortcoming that renders it difficult to use their KeTpic programs. To resolve this situation, KDG developed the KeTpic programming style for drawings in 2013. KeTpic programs include three main parts. Since 2014, KDG has improved KeTCindy as a plug-in for an interactive geometry software Cinderella. Cinderella has two screens: the interactive geometric screen and a screen called Script Editors. When a KeTCindy command is run in Script Editors, the corresponding KeTpic commands are extracted to the proper position of the three parts in a Scilab executable file.

KDG produced the web site "Making of Teaching Materials with Figures by KeTCindy" which described the utilization of KeTCindy and examples of class materials produced by KeTCindy. This web site is convenient for mathematics teachers when they create an original class material by KeTCindy.

In order to evaluate the effect which these figures in mathematics class materials give on the understanding of the students, KDG developed CDC (Cognitive Detection Clicker) which records the students’ answers in chronological order. When KDG carried out an experimental class using class materials produced by KeTCindy, the transition of students' answers was measured by CDC. The author introduces this measuring results.

43**. Lilla Korenova**

GeoGebra in elementary education

GeoGebra in elementary education Mathematics in elementary schools is focused on building basic mathematical literacy and developing cognitive areas - knowledge (facts, procedures), application (using the gathered knowledge to solve real-life problems), reasoning (solving more complex problems, that require a broader view of connections and relationships). The whole process of gathering new mathematical knowledge has to be carried out by the teacher, primarily with the aid of observation and experimentation in the pupils' natural environment. Learning mathematics has to be interesting, so they form a positive relationship towards it and to perceive mathematics as a tool to solve problems from everyday life. To achieve these goals, digital technologies are being used recently. Mostly interactive whiteboards, mobile technologies such as tablets or notebooks. GeoGebra is free software that has endless applications in teaching mathematics, from elementary education, up to university studies. In this paper, we would like to point out some of the possibilities of GeoGebra in elementary education in different contexts. At the same time, we would like to share our experience on how GeoGebra and the methodics of its use got accepted by teachers during continuous education and by future teachers - students of Faculty of Education, Comenius University in Bratislava.

44. **Pedro Quaresma**

Intelligent Geometry

The pursue of an "Intelligent Geometry Book" should involve the study of how current developing methodologies and technologies of mathematical (e.g. geometry) knowledge representation, management, and discovery can be incorporated effectively into the education of the future. To succeed in that we must increase the connections between several research communities: mathematical knowledge management; computer theorem proving: education.

Such an Intelligent book for Geometry, should be available in all standard computational platforms and devices, collaborative, adaptive to each and all users' profiles. A "book", i.e. a Cloud platform, capable of formal deductions, with a natural and visual languages interface, supporting a community of users with access to repositories of geometric knowledge and remote computational servers,

To realize such book the aggregation of many expertise is needed in the areas of "Proofs in a Learning Context"; "Interfaces & Searching"; "Tools Integration; "Learning Environments in the Cloud".

We will describe a working plan aiming at that goal.

45. **Joris van der Hoeven**

GNU TeXmacs as a CAS front-end

GNU TeXmacs is a free wysiwyw (what you see is what you want) platform for editing scientific documents. Its development started in the late nineties and the latest version is available from http://www.texmacs.org. TeXmacs provides a unified and user friendly framework for editing structured documents with different types of content such as text, mathematics, computer algebra sessions, graphics, animations, hyperlinks, spreadsheets, etc. The rendering engine uses high-quality typesetting algorithms for the production of professionally looking documents, which can either be printed out or presented from a laptop. TeXmacs runs on all major Unix platforms, Mac OS X, and Windows.

Some parts of TeXmacs were originally inspired by TeX and LaTeX. However, contrary to other programs such as LyX or Scientific WorkPlace, TeXmacs is not a graphical front-end for LaTeX, and an alternative rendering engine has been rewritten from scratch in C++. Besides an improved typesetting quality with respect to TeX, the rendering engine has the major advantage that documents are typeset in real time. This makes it possible to edit documents in a wysywig and user friendly way, without being distracted by compilation issues or encrypting formulas by LaTeX code.

Another objective of TeXmacs is to promote the development of free software for and by scientists, by significantly reducing the cost of producing documents, presentations, but also high quality user interfaces with other software. TeXmacs currently supports interfaces for many free computer algebra systems, such as Axiom, Macaulay 2, Mathemagix, Maxima, Pari, Reduce, Sage, etc., for several other mathematical systems, such as Octave, Scilab, GNU R, Graphviz, TeXgraph, etc., and for certain versions of a few proprietary systems, such as Maple and Mathematica.

External systems for doing computations can be invoked in various ways:
1. The most classical communication is based on shell-like sessions, in which it is possible to evaluate commands and display the results in a nice, graphical way.
2. The external system can also be used as an aid for editing documents. For instance, one may use it to differentiate or simplify the current formula or the current selection.
3. A recent new feature (under development) is a spreadsheet facility, where any computer algebra system can in principle be used as a spreadsheet language.

Different systems can also be combined in a natural way. For instance, results computed by one system can be copied and used as input for another system.

Using the above mechanisms, TeXmacs makes it easy to write highly interactive documents: computations are directly embedded into the document and the reader can experiment with alternative inputs. Spreadsheets can either be presented in the form of tables, as in other software, or in the form of individual labeled fields that are embedded into the text. The latter style allows users to create interactive exams in which solutions can be checked or computed automatically and for which the inputs might be generated at random.

46**. Partová Edita**

Diagnostics and the development of geometric knowledge through a variety of constructing tools

Geometry is the oldest area of mathematics that has been systematically elaborated theory also applied in practice. In school education students learn geometry over abstract definition, therefor we can found underachievement of their understanding of geometrical concepts and constructions. During the preparation of teachers of primary education, we seek ways to deepen and widen basic knowledge of geometry. We used it in addition to the classic construction tools also alternative devices, including the paper folding, dynamic geometry software and objects of everyday life eg. pins and string. Like paintings drawn in the sand, hand-painted and the paint program created image requires different knowledge creator, it is similar with geometric design alternative devices. The paper gave the example of the use of those devices and analyzes their advantages in understanding geometric knowledge.

47. **Zsolt Lavicza, Mamdouh Soliman and Maryam Al-Kandary**

Improving students’ learning through technology integration in Kuwait

This two-year project aims to improve Kuwaiti students’ learning and motivation through carefully integrating technology into the teaching process in their schools. In the first phase of the project, we worked with teacher trainers and teachers to evaluate their conceptions and practices with technology use in schools. Based on this, we delivered workshops to introduce new approaches to technology use in their classes. The training and materials for this workshop was based on the Geomatech project, a large-scale EU funded project in Hungary involving material development and training in a close to a thousand Hungarian schools, and was adopted to the Kuwaiti requirements. Teachers and researchers then developed new resources and approaches to be trialled in their own classrooms. The data collection of the project followed a design experimental approach to aim to holistically evaluate all aspects of classroom technology practices. After the analysis of the first phase of the project new phases will follow with more calibrated technology integrations in classrooms. In our talk and paper we will outline the design of materials, approaches, and report on the first round of data analysis.

48**. Natalija Budinski and Dragica Milinkovic**

Learning mathematics trough real life situation with use of educational software

The availability of technology has a big impact on education, and that is the main reason that in our presentation we discuss the use of technologies in mathematical education. The availability of technology influenced on how mathematical contents could be presented to students. We present benefits of learning mathematical concepts trough real life situations and share experiences from Serbia and Bosnia and Herzegovina. Particularly, we analyze process of transition from real life to mathematical situation and vice versa trough classroom examples which gave more clear picture of mathematical importance to students and motivated them to explore benefits of mathematical theory with the technology.

49. **Natalija Budinski**

Geogebra as a tool for connecting Materials Science and high school Mathematics

Materials Science is a branch of science which is exploring different materials, such as rubber, plastic, wood and many others that are used in everyday life. Despite its importance, it is not widely recognized. This presentation will introduce audience how scientific concepts from this science can be conveyed trough high school mathematics with help of Geogebra. Students can explore mathematical function properties while explore properties of materials. This new approach is beneficial to students because it picture how to connect mathematics, technology and high science and can help students to determine their future in those fields.

50. **Natalija Budinski**

Geogebra and origami-connection between technology and hands-on activities

The ancient Japanese art of Origami can contribute that students' better understand of mathematical concepts. In our educational approach we combine origami and Geogebra and offer examples on how to use them in mathematics classrooms. We propose the use of dynamic mathematics software GeoGebra with origami. Our approach we combine problem solving by using hands-on models as well as in a digital environment. With this combination, the two approaches supplement each other. In our presentation, we will offer examples on how to combine Origami and GeoGebra in teaching perpendicularity, Pythagorean theorem, doubling the cube problem, and polyhedrons. We will share results of this study in our presentation together with engaging folding and digital problems.

52. Kristóf Fenyvesi, Zsolt Lavicza, Diego Lieban, Imre Nyögéri, Ho-gul Park & Taeyoung Choi

STEAM Workshops for Collaborative Problem Solving Based on Connecting Hands-on 4dframe Activities with the Implementation of Geogebra

A construction system and educational building set, the 4Dframe was developed by Ho-Gul Park, a Korean engineer and model maker originally inspired by classical, Korean architecture. 4Dframe's concept is based upon the structural analysis and geometric formalization of building techniques utilized in the construction of Korea's traditional, wooden buildings. The set itself consists of just a small number of elegantly structured, simple module pieces. The wealth of structural variability offered by this versatile device renders it an excellent educational tool for conceptualizing, modelling, or analyzing topics relevant to science, technology, engineering (including robotics), arts (including architecture, or design), and mathematics. Due to its numerous advantages, the 4Dframe is perfectly adaptable to a wide variety of educational uses (Park, 2006) related to phenomenon-based learning and to the STEAM (Science, Technology, Engineering, Art & Mathematics) approach (Ge et al., 2015)
The central aim underlying 4Dframe educational methodology (Manninen, 2010) is to activate students' familiarity with geometric structures, within the context of problem-solving. This approach is based upon the creative exploration of these structures, attained through the step-by-step, scientific analysis of each stage in the construction process. The 4Dframe set is an effective tool that can be used to demonstrate and actively analyze any variety of geometric structures, problems, from planar tessellations to complex spatial structures.
The 4Dframe set was made in polypropylene, a material not only flexible enough for the construction of "unbreakable" modules, but also appropriate for inexpensive mass-production. The tubes included in the basic set are 2-30 centimetres in length; but to fulfill individual requirements, a pair of scissors is all that is needed to adjust their size. At the same time, a slit can be made into tube's opening in order to adjust the tube's diameter, thereby making it possible to use each tube as a connecting piece. While the set contains various types of connectors, these can also be easily adjusted, opening the door to an infinite number of creative solutions. The 4Dframe system's high degree of variability makes it the perfect medium for the modelling of any type of geometric construction or structure. (Park, 2015).
In this paper, we will introduce several examples for collaborative problem solving STEAM workshops connecting hands-on 4dframe activities with the implementation of Geogebra software.

53**. Hunor Nagy and Pál Kupán**

Improving the students performance using digital tools in geometry instruction

In this talk I'd like to speak about teaching methods that use digital tools in the middle school (6 th – 7 th - 8 th class for 12 – 13 - 14 years students).
I use traditional and alternative methods in teaching geometry, but I encountered problems in the 6th class when I teach how to edit geometric figures. There was also problems in 8 th class regarding space geometry. It was difficult for the children whose mathematical knowledge is low to middle to edit fair figures. There was also problems with the spatial vision. To improve the results of the students I use GeoGebra. I work with this method and I'd like to talk about this experience.
I mention that it is a new method in our school and our country. So I hope I give new horizons to our students.