Introduction. As the content of education is realized via curriculum subjects, it is a paramount task to build the coordinated system of academic subjects structured with preserved links between elements of knowledge. On the other hand, any academic subject, especially in pre-service Maths teachers’ training, is an embodiment of some scientific domain adapted to teaching and learning. Hence, it must reflect correctly and adequately the structure of knowledge domain, preserving main links between notions, concepts, facts, theories that really exist both inside the scientific branch and between sciences in a whole. It will promote and contribute to creation holistic and flexible system of pre-service teachers’ knowledge and their readiness to form similar system in their pupils’ minds in future. However, very often the modular structuring of curriculum subjects does not preserve or does not convey necessary essential links between scientific knowledge what can cause negative consequences for trainees’ (forming of separate and uncoordinated system of trainees’ knowledge, acquisition of purely specific skills instead of generalized ones, breaking of general wholeness and logic of a subject as well as destroying of links between related subjects etc.) So, the central problem is how to form integrative content of learning, how to create interdisciplinary coordinated curriculum exactly for Math teachers’ training, and how to provide modular structuring of curriculum subjects with preserving links between knowledge.

Objectives of the workshop:
1) to discuss and represent knowledge integration as a result of natural integration tendencies in science and their influence on the structure and content of academic disciplines; to highlight necessity of knowledge integration reflection in the system of curriculum subjects in Math teachers’ training;
2) to represent mechanisms of knowledge integration; to demonstrate author’s didactic and digital tools of their realization in the process of interdisciplinary curriculum development. The tools include special procedures and steps which are expected to be fulfilled by the participants of the workshop in the process of collaborative work upon the set of author’s case studies.
3) to discuss teacher’s tasks and activity under the terms of interdisciplinary approach to learning;
4) to discuss didactic problems of Maths teachers’ training which can be solved with the help of such an approach to the curriculum development.
I. Mechanisms of knowledge integration in scientific branches and in curriculum disciplines. In order to solve the problem of forming integrative learning content we have done special research that allowed us to elaborate so called basis of knowledge integration for curriculum disciplines structuring which includes some main theses:

1) knowledge integration by its essence is a penetration of knowledge from one scientific area to another, as a result knowledge of more information capacity appears. The conditions of knowledge integration are information exchange on both empirical and theoretical levels.

2) According to studies, process of knowledge formation is tightly connected with integrative processes in science: deep learning of the main subject of a science leads to necessity of attracting knowledge and methods of other sciences. On the other hand, permanent expanding of knowledge sphere requires compression of knowledge and sets the task of contraction of knowledge in order to make it more concentrated and visually graspable at the same time. Hence, dedifferentiation naturally leads to integration and encourages integrative tendencies in science.

3) In order to realize the influence of these objective integrative tendencies in science on education and on the formation of educational content in vocational education especially we revealed main aspects of knowledge integration influence from the standpoint of educational content: 1) origin (basis) of sciences interaction; 2) types, forms and means of inter-scientific links; 3) problems of monodisciplinarity and inter-disciplinarity. The analysis of these three aspects gave us understanding that knowledge integration in scientific branches is embodied in:

1) reflection of interactive processes in nature and society, i.e. scientific branches reflect natural interactive processes which take place in nature and society

2) intersection of science objects; in creation of unified conceptual mechanism; in extrapolation and extension of investigation methods of one discipline to another; in formation of complex investigation methods which concentrate knowledge on the same object from different sciences; in convergence of various forms of cognition typical for different branches.

3) in community of cognitive problems which are solved by the branches; in revealing such features of the science object which are common to the objects of other sciences; in the realization of the philosophical potential of the branch and its autonomy.

Using these results and the analysis of the process of transformation of a scientific branch into an academic discipline we determined some procedures which have to be realized by the didactic component of a curriculum subject in order to reflect scientific knowledge integration:

- revealing specific characteristics of the subject and measures of implementation of its conceptual and methodological arsenal;
- forming of fundamental all-over-scientific notion mechanism;
detecting integration potential of the subject, learning cross-discipline methods of research;
• providing adequate types and mechanisms of knowledge arrangement which are able to reflect variety and complication of cross-discipline links;
• Carrying out three-aspect mutual penetration of curriculum subjects via formed fundamental notion mechanism, cross-discipline means of cognitive activity, and information content of subjects.

These procedures actually play the role of the necessary mechanisms of knowledge integration.

II. Didactic and digital tools of realization of knowledge integration mechanisms in the process of interdisciplinary curriculum development. In order to make these mechanisms function we developed special didactic and mathematical tools which allow to get as a result the modular structure of curriculum subjects keeping and spreading links between knowledge both inside a module and between modules and subjects of curriculum.

These procedures are based on the ideas of the:
1) different levels of the education content formation;
2) different levels of knowledge generalization;
3) models of knowledge representation of the Artificial Intelligence theory.

Actually, models of knowledge representation play the role of mathematical tools for multilevel formation of the content of learning based on knowledge integration. In particular, we used semantic networks and frame-based systems as our main tools and their inheritance property which allows to realize links between elements of knowledge.

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Brief reference from Artificial Intelligence theory

Knowledge representation is the method used to encode knowledge in an intelligent system knowledge base. Knowledge is an abstract term to capture an individual’s understanding of a given subject. In the world of intelligent systems the domain-specific knowledge are considered. Here domain is a well-focused subject area, and knowledge is the set of facts, regularities, relations and rules which reflect the level of awareness on the problem of this domain. Among various models of knowledge representation the most useful from the standpoint of realization of knowledge penetration on different levels of its generalization seem to be semantic networks and frame representation.

Semantic nets. Author: Quillian, 1967
Idea: Concepts are a part of knowledge about world. People perceive concepts and reason with them. Concepts are related with relationships between them. Relationships between concepts form understanding of people.

Definition of semantic nets: Semantic network is a knowledge representation model that captures knowledge as a graph. The nodes denote objects or concepts, their properties and corresponding values. The arcs denote relationships between the nodes. Both nodes and arcs are generally labeled. In the context of our research is important that inheritance works in semantic nets. Inheritance is a process by which the local information of a superclass node is assumed by a subclass node, a subclass node, and an instance node.

Another really fruitful and relevant instrument to use at the discipline structuring is frame-based knowledge representation. Author: M.Minsky, 1975
Psychological Idea: When we encounter a new situation we select from our memory a structure called a “frame”. This is a remembered framework to be adapted to fit reality by changing details as necessary.
Definition of a frame: According to M. Minsky, a frame is a static data structure used to represent stereotyped situations. The psychological idea was used successfully in knowledge representation in such a way. Frames are considered to be packets of information about concepts or entities and their instances: all the information relevant to a particular concept is stored in a single complex entity, called a frame. Each frame has its basic structure which can be represented as a table:

<table>
<thead>
<tr>
<th>Slot Name</th>
<th>Inheritance Pointer</th>
<th>Data type</th>
<th>Slot Value</th>
<th>Demon Procedure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slot1</td>
<td>Frame1Name.Slot1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Frame1Name.Slot2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Frame1Name.Slot3...</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slot N</td>
<td>Frame2Name.Slotn...</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Each frame has one or more slots, which are assigned slot values. The most important element of each frame is Inheritance Pointer. It contains information about relationships between this frame and its slots and slots of other frames. As with semantic nets, a frame system consists of a set of frames (or nodes), which are connected together by relations. Each frame describes either an instance (an instance frame) or a class (a class frame).

The highest level of education content formation (and correspondently, of knowledge generalization) contains “super-discipline” general system of common knowledge, skills, abilities, ways of activity. This “super-discipline” content is concentrated as a network of “key-points” which are to be learnt in order to provide for a student unified image of reality. These “key-points” are defined basing on the analysis of a) the system of professional functions of a pre-service specialist; b) the structure of his activity; c) essence of his typical professional tasks. So, they include real objects, processes, phenomena which are to be mastered by per-service specialists. As a result we obtain on this step a semantic network which reflects “super-discipline” system of knowledge, skills, abilities, ways of activity (Fig.1). Around these “key-points” we will concentrate learning material of specific disciplines on the lower levels of education content formation.

![Image of network](image-url)

Fig.1. Fragment of the network, which represents “super-discipline” general system of knowledge, skills, abilities, ways of activity of a pre-service specialist

The second level of knowledge generalization (after “super-discipline” level) matches the level of the curriculum discipline.
Here general knowledge, skills, abilities, ways of activity (determined on the highest level) take more specific form and are spread among disciplines. This is so-called “structure” level of knowledge generalization. It provides general structure of the subject area and rules of knowledge manipulation. In order to realize it we elaborated special technique of preliminary analysis of the subject area and revealing four groups of didactic characteristics of academic disciplines:

The first group includes the subject, aims (of different levels) and tasks of the discipline; requirements to initial readiness of students necessary to succeed the discipline; academic hours and proportions between theoretical and practical part; target audience of the discipline.

The second group of characteristics actually covers its content: main concepts, methods, main problems of the discipline and their connections with contemporary state of science and practice.

The third group of the discipline didactic characteristics includes (1) its links with other disciplines of different cycles of professional training and the list of these links; (2) the list of common skills and abilities of students which the discipline is able to form and develop, and which are important for learning other disciplines (for example, abilities to analyse, to generalize, to classify, to model, to make hypothesis etc.); (3) areas of the discipline knowledge application.

The fourth group of the discipline didactic characteristics includes peculiarities of cognition process, main types of learning activities, specific technology of teaching, forms of control etc.

So, to provide a comprehensive analysis of the discipline we have to fill in all of these groups of the discipline characteristics. After that, on the second level of knowledge generalization we can select so-called “node” learning elements (Les). They are elements of learning material around which we should concentrate learning material at modular structuring. Here we also establish links between selected “node” LEs of the discipline and “node” LEs of other disciplines using the analysis of the third group of didactic characteristics. On the other hand, these selected “node” learning elements have to provide links with “key-points” of the “super-discipline” (the highest) level. So, they are grouped around these “key-points” of the “super-discipline” level. It is usually done in the form of the table:

<table>
<thead>
<tr>
<th>Key-points of “Super-discipline” level</th>
<th>Learning Elements of the level of a discipline</th>
</tr>
</thead>
<tbody>
<tr>
<td>KP1</td>
<td>LE1, LEi,...LEj..</td>
</tr>
<tr>
<td>KP2</td>
<td>LEk, LEi,...LEj..</td>
</tr>
<tr>
<td>....</td>
<td></td>
</tr>
<tr>
<td>KPn</td>
<td>LEk, LEi,...LEj..</td>
</tr>
</tbody>
</table>

These procedures allow to make “rough” modular structuring of the discipline with establishing necessary links between LEs inside each modules and between modules of this and other disciplines. So, these procedures cover the second level of education content formation, the second level of knowledge generalization and they correspond to the first basic procedure of frame-based knowledge representation which
provides general structure of frames system with their names and established links. As a result we obtain hierarchy structure with frames-prototypes in its nodes, where each frame corresponds to cycles of training (C), curriculum subjects (CS) and to their modules (M) with learning elements (LE) (Fig.2, p.11).

The third level of knowledge generalization corresponds to the level of learning material. Actually on this level we provide next basic procedure for frame-based knowledge representation: we fill in all frames-prototypes of general structure and their slots with exact data-exact values of LEs, exact Inheritance pointers for each slot to provide proper links.

As a result the modular structure of the subject built in such a way provides keeping and spreading links between knowledge both inside a module and between modules and subjects of curriculum. More over such a structure is sensitive to the changes of the curriculum and to the social demand.

The next stage of the structuring is an expertise according to special criteria as it is necessary to estimate the quality of the built structure before introducing it into practice. It might be the topic of separate workshop.

The set of Case Studies

Case Study 1. Let us assume that we are creating educational content and interdisciplinary curriculum for pre-service Mathematics teachers using presented technology of multilevel education content formation and multilevel knowledge generalization.

The task to Case Study 1 is to determine key-points of the highest ("Super-discipline") level of knowledge generalization and content of education respectively, and put them into the Table 1.1:

<table>
<thead>
<tr>
<th>Key-points of the highest level</th>
<th>The essence of Key-points of the highest level of educational content for pre-service Mathematics teachers</th>
</tr>
</thead>
<tbody>
<tr>
<td>KP1</td>
<td></td>
</tr>
<tr>
<td>KP2</td>
<td></td>
</tr>
<tr>
<td>KP3</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
</tr>
<tr>
<td>KPn</td>
<td></td>
</tr>
</tbody>
</table>
Case Study 2.1. Let us assume that we are building the modular structure of the curriculum subject “Discrete Mathematics” of our interdisciplinary curriculum for pre-service Mathematics teachers.

The tasks to Case Study 2.1. are:

to cover four groups of didactic characteristics for Discrete Mathematics discipline (Table 2.1.1.);

to select “node” learning elements of the subject around which we should concentrate learning material at modular structuring;

to match selected “node” learning elements and the key-points of the highest level of knowledge generalization determined in the Case Study 1 (to fill in the table 2.1.2.).

Four groups of didactic characteristics for Discrete Mathematics discipline

<table>
<thead>
<tr>
<th>Group Number</th>
<th>Didactic characteristics of the group</th>
<th>The essence of didactic characteristics of the group</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-the subject, -the aims of different levels: to get acquainted with...; to know...; to have skills of...; -tasks of the discipline;</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Prerequisites as for initial readiness of students necessary to succeed the discipline;</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>academic hours and proportions between theoretical and practical part;</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>target audience of the discipline</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>main concepts;</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>methods;</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>problems and their connections with contemporary state of science and practice.</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>The list of links with other disciplines;</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>the list of common skills and abilities of students which the discipline is able to form and develop that are important for learning other disciplines;</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>areas of the discipline knowledge application.</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>peculiarities of cognition process, main types of learning activities, specific technology of teaching, forms of learning control</td>
<td></td>
</tr>
</tbody>
</table>
Matching “node” Learning Elements of the discipline and the Key-points of the highest level of knowledge generalization determined in the Case Study 1

<table>
<thead>
<tr>
<th>Key-points of highest level of knowledge generalization</th>
<th>The essence of Key-points of the highest level of educational content for pre-service Mathematics teachers</th>
<th>“Node” Learning Elements of the level of a discipline</th>
</tr>
</thead>
<tbody>
<tr>
<td>KP1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>KP2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>KP3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
</tr>
<tr>
<td>KPN</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Case Study 2.2. Let us assume that we have done the tasks of Case Study 2.1 for the curriculum subject “Discrete Mathematics” of our interdisciplinary curriculum for pre-service Mathematics teachers.

The task to Case Study 2.2.: Basing on the results of Case Study 2.1, obtain hierarchy structure with frames-prototypes in its nodes where each frame corresponds to disciplines and to their modules (similar structure is shown in Fig.2, p.11).

Case Study 2.3. Analyze the results of Case Studies 2.1 and 2.2, and use them to fulfill the task.

The task to Case Study 2.3 is: to accomplish building of frame-based structure of the curriculum subject “Discrete Mathematics”. Establish proper Inheritance Pointers, add frames which correspond Modules (Ms) and selected “node” Learning Elements which are grouped into Modules.

III. Teacher’s tasks and activity under the terms of interdisciplinary approach to learning

Case Study 3. Let us assume that we are going to use obtained modular structure of the curriculum subject “Discrete Mathematics” on the level of teaching and learning activity.

The task of Case Study 3 is:
- to select any module of the discipline,
- to determine its didactic goals,
using frame-based modular structure and established links between Learning Elements, to pick up integrative learning activity for students (chain of questions on links restoring; practical tasks on application of knowledge from other units, modules, and subjects; interdisciplinary projects etc.)

IV. Discussion of didactic problems of pre-service Maths teachers’ training which can be solved with the help of such an approach to the curriculum development.

Questions

1. What are the benefits of the offered technology of disciplines structuring from the standpoint of the University teacher who trains pre-service Math teachers?

   It allows a teacher:
   - to see and realize the place of each Learning Element in the system of learning elements of this discipline and its links with other disciplines via mutual penetrating, and hence
   - to select proper methods and techniques of learning,
   - to offer students interdisciplinary problems to solve,
   - to provide special ways of students’ activity in order to form their holistic and flexible system of knowledge and skills.
   - to save academic time, and at the same time to learn each learning element comprehensively, deeply and from the standpoint of various disciplines.

2. What is practical application of coordinated and consistent modular structuring of subjects on all levels of formation of education content?

   It might be used
   - for development of adaptive learning systems,
   - for innovation curriculum programs creating with interdisciplinary content,
   - for effective tests creating.

3. What are prospective social benefits of the presented technology of disciplines structuring and interdisciplinary curriculum design?

   The main benefits are:
   - forming students’ holistic knowledge system of optimal information capacity capable to be used flexibly in related branches;
   - automatic control of cognitive processes in education;
   - creating optimal educational trajectory from the standpoint both of a student and the situation on labour-market;
   - determination of equivalence degree of related specialities and as a result - facilitating public professional and social mobility;
   - education services delivery arrangements;
   - developing optimal hybrid forms of education provision and governance; promoting searching adequate mechanisms of widening range of educational service regulation and others.

Main results are published:

- Gryzun L.E. Information technology of the subject modular structure design based on scientific knowledge integration : [mode of access] http://www.journal.iitta.gov.ua
- Gryzun L.E. Integrative technology of academic subjects structuring and its applications to practical didactic issues http://drohobych.net/youngsc/AQGS