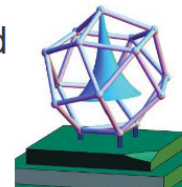




Sixth Central- and Eastern European  
Conference on Computer Algebra- and  
Dynamic Geometry Systems  
in Mathematics Education

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# Obstacles in combining the use of various tools in solving mathematical problems – why is Copy/Paste often useless

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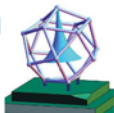
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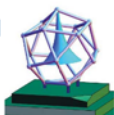
# CADGME 2014

- Talk *Lessons Learned in Course Computer Tools In Mathematics*
- We talked (also) about
  - One tool covering all aspects of teaching and learning mathematics, or many "smaller" tools
  - Interoperability of tools



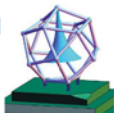
# Course *Computer Tools In Mathematics*

- Practical Mathematics
  - First cycle professional study program
- Emphasis on hands on solving "old mathematical tasks" (from high school) with various tools (Mathematica, GeoGebra, Octave, Excel, Numpy ...)
- **Subject specific competences developed by the student:**
  - ability of employment of mathematical tools at practical problem solving,
  - ability of result analysis,
  - ability of **presentation of results**
  - ...



# Experiment

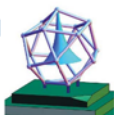
Having chosen different widespread software programs, students explored transfers of mathematical object between them using copy and paste.



# Background

- In the first weeks in the course *Computer tools in mathematics* students get used to **some typical examples of mathematical software**.
- They obtained mostly **just the basic knowledge**, so they know only the most common functions and functionality of the used applications
- **Experiment: To solve** (and **to report** in detail the process of obtaining the solution) a certain mathematical task
- The usage (within the same task) of **different mathematical software** has been foreseen

**Had to report on all difficulties and obstacles**

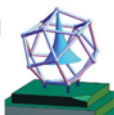


# Example

*Given are points  $A$ ,  $B$ ,  $C$  and  $D$ . Construct a cubic polynomial  $p(x)$  through those points. Use GeoGebra to draw it. Select arbitrary point  $(x, p(x))$  and draw a tangent to the graph. To calculate the polynomial, use suitable procedure in OCTAVE.*

*Report about the process, describe steps, justify your findings*

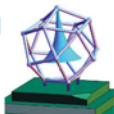
*Report also about the obstacles, difficulties and your impressions when doing this task!*



# Exchange of mathematical objects

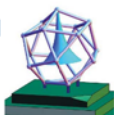
- Work with several systems of mathematical software
- Reporting

Copy / Paste procedure is expected to be used by students



# Copy and paste from the technical point of view

- An origin „place“ is selected (e.g. with the mouse)
- It is copied
  - several copies of the content into the system's clipboard,
  - one for each flavor, in order of preference.
- A target place is selected in a document being edited.
- The contents of the clipboard is pasted.
  - The application editing that document chooses the best flavor
  - inserts its associated content at the given place.

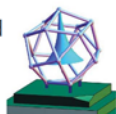
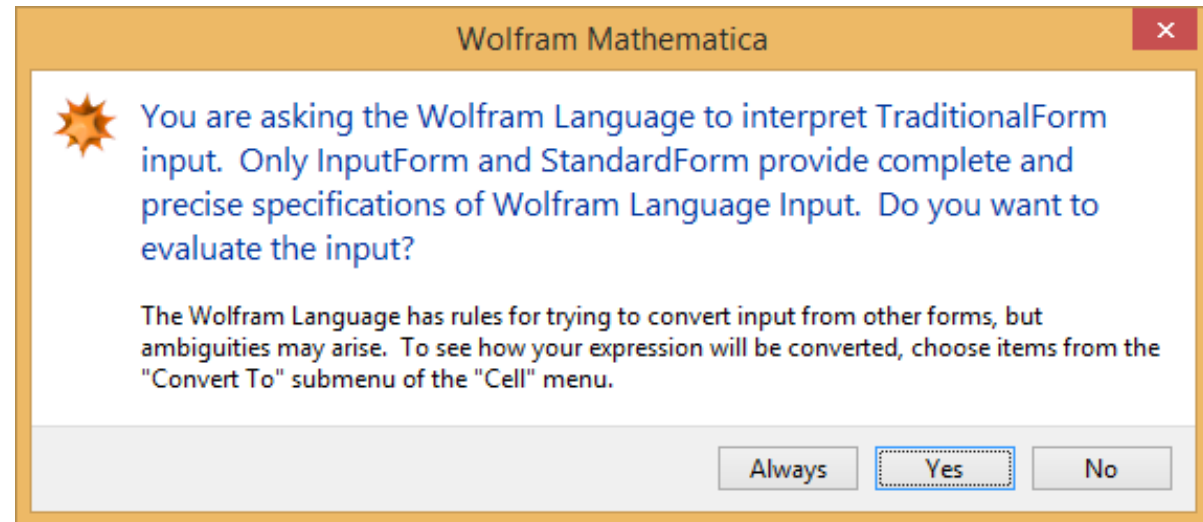




Some „real life“ cases

# From Word to Mathematica

- *Word (Equation mode)*
  - *Insert > Equation*
  - marking the expression
  - Ctrl-C (the copy operation)
- Mathematica
  - Ctrl-V (Paste)



# Many examples perform well

$$\text{In}[1]:= \sum_{n=1}^{\infty} \left( a_n \cos \frac{n \pi x}{L} + b_n \sin \frac{n \pi x}{L} \right)$$

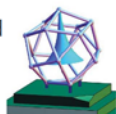
$$\text{Out}[1]= \sum_{n=1}^{\infty} \left( \cos \left[ \frac{n \pi x}{L} \right] a_n + \sin \left[ \frac{n \pi x}{L} \right] b_n \right)$$

$$\text{In}[2]:= \frac{\frac{2}{a-b}}{3 + \frac{1}{a-b}}$$

$$\text{Out}[2]= \frac{2}{\left( 3 + \frac{1}{a-b} \right) (a-b)}$$

$$\text{In}[3]:= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\text{Out}[3]= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



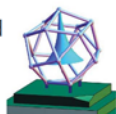
# But ...

```
In[16]:= 2 • 3
```

```
Out[16]= 6 •
```

```
In[29]:= sen (2 π)
```

```
Out[29]= 2 π sen
```



# What about text mode ...

$2 + 3 - a + b - a$

In[1]:=  $2 + 3 - a + b - a$

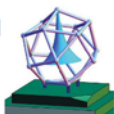
Out[1]=  $5 - 2a + b$

- Problems with auto format
  - 6-b becomes  $6 - b$  (*dash!*)
  - $1/4$  becomes  $\frac{1}{4}$

$2 + 3 - a + b - a$

In[2]:=  $2 + 3 - a + b - a$

Out[2]=  $5a(a + b) - 2$



# From Mathematica to Word

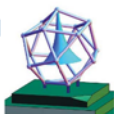
- Problems with peculiarities of Mathematica's syntax like [] for functions' arguments
- Word –Mathematica - Word

$$\text{In}[2]:= a_0 + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n \pi x}{L} + b_n \sin \frac{n \pi x}{L} \right)$$

$$\text{Out}[2]= a_0 + \sum_{n=1}^{\infty} \left( \text{Cos} \left[ \frac{n \pi x}{L} \right] a_n + \text{Sin} \left[ \frac{n \pi x}{L} \right] b_n \right)$$

$$a_0 + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n \pi x}{L} + b_n \sin \frac{n \pi x}{L} \right)$$

$$a_0 + \sum_{n=1}^{\infty} \left( \text{Cos} \left[ \frac{n \pi x}{L} \right] a_n + \text{Sin} \left[ \frac{n \pi x}{L} \right] b_n \right)$$



# GeoGebra and Mathematica

- GeoGebra CAS mode!

The screenshot shows the GeoGebra CAS mode interface. The top menu bar includes File, Edit, View, Options, Tools, Window, and Help. Below the menu is a toolbar with various mathematical symbols and functions. The main workspace displays three input-output pairs:

	Input	Output
1	$2 + 3 - a + b$	$-a + b + 5$
2	$2a - 3b + a c$	$a c + 2 a - 3 b$
3	$abc - 3 a + abc$	$-3 a + 2 abc$

The screenshot shows the Wolfram Mathematica 10.0 interface. The top menu bar includes File, Edit, Insert, Format, Cell, and Graphic. The main workspace displays two input-output pairs:

	Input	Output
1	$a + 3b + a$	$2 a + 3 b$
2	$a + 3b + a$	$2 a + 3 b$

In[1]:=  $(2 * \alpha) + (3 * \beta)$

Out[1]=  $2 \alpha + 3 \beta$

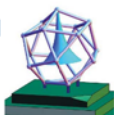
In[2]:=  $\alpha + 3 \beta + \alpha$

Out[2]=  $2 \alpha + 3 \beta$



# But in opposite direction

CAS	
In[1]:= $\alpha + \beta$	
Out[1]= $\alpha + \beta$	$\backslash[\text{Alpha}] + \backslash[\text{Beta}]$
	→ Invalid input





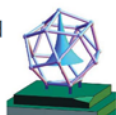
# Also with Copy as ...

2

```
<math xmlns='http://www.w3.org/1998/Math/MathML'   mathematical:form='StandardForm'   xmlns:mat
```

→ One or more references are invalid

Copy as MathML



# Free form input In Mathematica

```
In[6]:= (-bla) + blo
```

```
Out[6]:= -bla + blo
```

```
In[7]:= = (-bla) + blo
```

(no interpretations available) ?

+

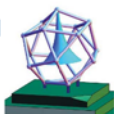
```
In[9]:= = (angle + fi) - frequency
```

↩ angle fi ?

↪ Last close

```
Row[{FinancialData["L:AGL"]
```

```
Out[9]:= $Failed$Failed
```



# Commands with (almost) the same name, but

...

1	<code>Factor[x^2 - 4]</code> → $(x - 2)(x + 2)$
2	<code>Factor[x^2 - y^2, x]</code> → $(x - y)(x + y)$
3	<code>Factors[x^2 - 4]</code> → $\begin{pmatrix} x - 2 & 1 \\ x + 2 & 1 \end{pmatrix}$
4	<code>Factors[2324]</code> → $\begin{pmatrix} 2 & 2 \\ 7 & 1 \\ 83 & 1 \end{pmatrix}$

```
In[10]:= Factor[x^2 - 4]
```

```
Out[10]= (-2 + x) (2 + x)
```

```
In[11]:= Factor[x^2 - y^2, x]
```

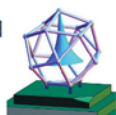
```
Out[11]= Factor[x^2 - y^2, x]
```

```
In[12]:= Factors[x^2 - 4]
```

```
Out[12]= Factors[-4 + x^2]
```

```
In[13]:= Factors[2324]
```



```
Out[13]= Factors[2324]
```



# Using FreeForm



Untitled-1 \* - Wolfram Mathematica 10.0



File Edit Insert Format Cell Graphics Evaluation P


In[2]:=  Factor[x^2 - y^2] 

Factor[x^2 - y^2]

Out[2]= (x - y) (x + y)



In[3]:=  Factor[x^2 - y^2, x] 

 x^2 - y^2 

 Roots for the variable y (1 of 2)



Solve[x^2 - y^2 == 0, y]

Out[3]= {{y -> -x}, {y -> x}}

In[4]:=  Factors[x^2 - 4] 

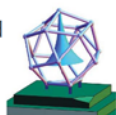
Factor[-4 + x^2]

Out[4]= (-2 + x) (2 + x)

In[5]:=  Factors[2324] 

Factor[2324]

Out[5]= 2324



# Matrices from Octave to Mathematica

```
Command Window
>> A=[1,2,3]
A =
     1     2     3
>> |
```

```
Wolfram Mathematica | STUDENT EDITION

In[2]:= A = [1, 2, 3]

Syntax::sntxf : "A =" cannot be followed by "[1, 2, 3]".

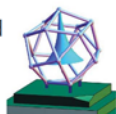
+

Choose how to enter input
```

```
>> B=[1 2 3;4 5 6; 7 8 9]
B =
     1     2     3
     4     5     6
     7     8     9
```

```
In[2]:= B= [1 2 3;4 5 6; 7 | 8 9]

Syntax::sntxf: "B=" cannot be followed by "[1 2 3;4 5 6; 7 8 9]".
Syntax::sntxf: "1 2 3;4 5 6; 7 " cannot be followed by "8 9]".
```



# Or using defined vector/matrix

Command Window

```
>> A=[1,2,3]
A =

     1     2     3

>> B=[1 2 3;4 5 6; 7 8 9]
B =

     1     2     3
     4     5     6
     7     8     9

>> |
```

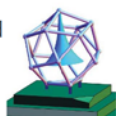
Wolfram Mathematica | STUDENT

```
In[7]:= A = 1 × 2 × 3
Out[7]= 6

In[8]:= B = 1 × 2 × 3
         4 × 5 × 6
         7 × 8 × 9
Out[8]= 6

Out[9]= 120

Out[10]= 504
```



# Operations

```
>> A+B
ans =

     5     7     9
     5     7     9
    14    16    18

>> A-B
ans =

     3     3     3
    -3    -3    -3
     0     0     0

>> A*B
ans =

    66    81    96
    30    36    42
   102   126   150

>> A'
ans =

     4     1     7
     5     2     8
     6     3     9

>> A./B
ans =

    4.00000    2.50000    2.00000
    0.25000    0.40000    0.50000
    1.00000    1.00000    1.00000

>> |
```

```
In[25]:= A + B
Out[25]= {{5, 7, 9}, {5, 7, 9}, {14, 16, 18}}

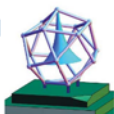
In[19]:= A - B
Out[19]= {{-3, -3, -3}, {3, 3, 3}, {0, 0, 0}}

In[26]:= A * B
Out[26]= {{4, 10, 18}, {4, 10, 18}, {49, 64, 81}}

In[21]:= A '
Out[21]= {{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}'

In[27]:= A ./ B
```

Syntax::snbxf : "A." cannot be followed by "/"B".



Observations and remarks



*“... Copy/Paste method does not work with drawing plots in different programs. For drawing plots each one of used programs has its own ‘language’, which we have to use. “*



*“When we copied expressions from Word to GeoGebra, it worked in some cases even without changing expressions. If exponents would be written with symbol ‘^’ in Word, there would not be any problems. From there on, some changes and adaptations were necessary, especially when copying expressions from GeoGebra to Mathematica”*



*“... I was very rarely able to use Copy/Paste  
method to transfer expressions between programs  
and even then some corrections were usually  
needed for programs to work”*



*“After several tries I gave up. The only way to transfer an expression between programs is to manually retype it ...”*



*“To input matrix in  $X$  is surprisingly identical to input matrix in  $Y$ . But unfortunately here the joy ends.”*



*“It has been an extremely tough task. I spent numerous hours doing it. I tried several techniques to force the program to properly copy/paste the expression with no success. Finally, I manually transferred the expressions. But here I had problems again as even the usage of parenthesis not even mentioning the names of the commands was not the same. I do not understand why software producers each one uses their own syntax.”*



*“Please do not give us such tasks anymore. It is impossible. All the time switching between different syntax. At the end I did not even know, if I got everything right, even though the mathematics behind was luckily simple enough.”*



And finally



*“it is all mathematical software and mathematics is just one, so I expected no problems in using Copy/Paste”*

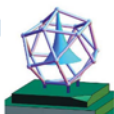
In the discussion afterwards they all express their deep disappointment with the software. When started the task, they mostly all expected there will be no problems or just some minor problems.

# Multiple issues

- Loss of information at copying
- Preference for pasting plain text when other formats may be more appropriate
- Misinterpretation of the mathematical notation
- Lack of usage of more semantic encodings
- Partial but unpredictable success

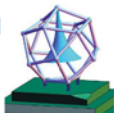
# Loss of information at copying

- some programs transfer much of the information into the clipboard
  - allowing other programs to recover most information (in principle),
- many put information in private encodings
  - difficult for other applications to receive in a consistent manner on the long term.



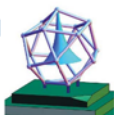
# Preference for pasting plain text when other formats may be more appropriate

- many programs ignore content types
  - that could bring more information
  - expecting that plain text is probably a better guess than, MathML, RTF, or HTML flavors



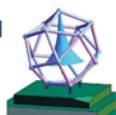
# Misinterpretation of the mathematical notation

- input of mathematical formulæ leaves the user quite free to use mathematical notations in different ways
- usage of the cross-sign for multiplications (fails in C/P Word to Mathematica).
- different software use different function names which are not properly exchanged



# Lack of usage of more semantic encodings

- Most mathematical software systems include a rich set of mathematical operators and functions.
- interoperability they offer
  - linear syntax (known to be very program-specific)
  - presentation-oriented syntax (such as MathML presentation or pictures).



# Partial but unpredictable success

- very diverse set of interoperability issues.
- no way to explain rules to tell when a formula will be transferred successfully.

