“What if not” investigation method using Geogebra for a geometric configuration of quadrilaterals that through a dynamic process aspire to be a square

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Abstract

The purpose of this article is to describe the building of a geometric configuration and explore it, in order to train students in teaching mathematics according to the “What if not?” (WIN) strategy, suggested by Brown & Walter (1990).

We consider the following basic principles of the construction of the configuration:

1. The configuration has a generalization potential (W.I.L.-“what if less?”, W.I.M. – “what if more”, W.I.I. – “what if instead”).
2. It has elements of “prove” and “calculate”.
3. It is challenging but not frustrating for students.
4. It allows construction of hypotheses which can be checked with Geogebra.
5. It was thoroughly researched by us to prevent divergence or dead ends.

The following example of configuration explains these principles:

Given parallelogram ABCD. Its external bisectors form quadrilateral MNPQ (Fig.1). What can one say about MNPQ?

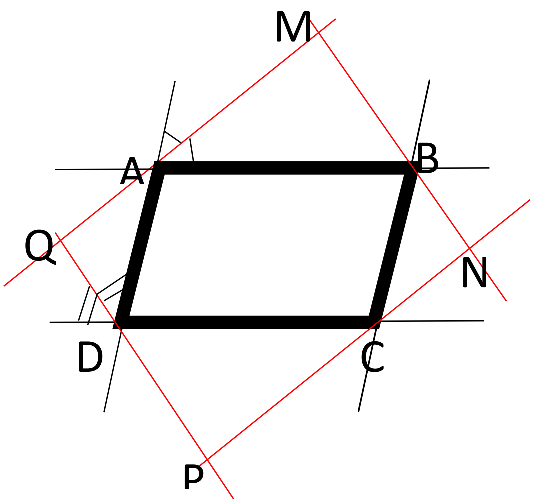

Fig.1

The configuration corresponds to the principles above:

1. It has a generalization potential:

- **W.I.M.** - What if ABCD is: a square? A rectangle? A rhombus? → **W.I.L.** - a kite?
- **W.I.L.** - What if ABCD is: a trapezoid? An isosceles trapezoid? An arbitrary quadrilateral? → **W.I.M.** - $\angle A = \angle C$ only?
- **W.I.I.** - What if ABCD is**:** a triangle? A pentagon?
- **W.I.I.** - What if we are given the internal bisectors?
- **W.I.L.** - What if the angles are divided in ratio that is not 1:1?

2. It has elements of "prove" and "calculate":

- Prove: the answers to all questions above demand proofs.
- Calculate: it is worthwhile to calculate the elements of ABCD and MNPQ (sides, angles, diagonals and etc.). The calculations help to "discover" properties of MNPQ depending on the properties of ABCD.

3. It is challenging but not frustrating for students:

   Students can come to an important generalization: quadrilateral MNPQ has a "higher rank" than ABCD. And generally there is a "promotion" in the transition from a quadrilateral to its successor.

4. It allows a construction of hypotheses which can be checked with Geogebra:

   We prepared three applets for involvement of Geogebra in constructions of hypotheses:

- Two applets for generalization "From a parallelogram to an arbitrary quadrilateral".
- One applet for generalization "From a quadrilateral to a triangle".

5. The configuration was thoroughly researched by us to prevent divergence or dead ends.

   We, as teachers and researches, should investigate the problem until the end as far as possible.

   Quadrilateral MNPQ is more "advanced" than ABCD:

- Parallelogram → rectangle ;
- Rectangle → square;
- Arbitrary quadrilateral → cyclic quadrilateral and etc.

   It is possible to prove an interesting fact: if we continue this process, all angles reach 90^o and quadrilaterals become a square.

Summary

Didactic field

- Students of a pedagogical college feel they do mathematical research.
- There is a chance that they will use this method (“What if not” investigation method) in working with their pupils at school.
- Students get more motivation for their professional development

Mathematical field

We found a lot of interesting and new (for us and maybe not only for us) properties and laws related to the considered configuration:

- Formulas - diagonals, perimeter, radius of circumcircle of MNPQ, etc.
- Quadrilateral ABCD has the smallest perimeter among all quadrilaterals inscribed in MNPQ (the quadrilateral of a ray of light).
- The fact that all angles approach 90° also if we start from another kind of polygon that is not quadrilateral.
- There are another properties and laws that we obtained as hypotheses and don’t know (for now) whether they are true.

REFERENCES

1. Brown, S., & Walter, M. (1990). “The art of problem posing (2nd ed.)”. Philadelphia: Franklin Institute Press.
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